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Question Paper Code : 40875

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2025.

Fourth Semester

Computer and Communication Engineering

MA 8451 – PROBABILITY AND RANDOM PROCESSES

(Common to: Electronics and Communication Engineering
Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $P(A)=0.4$, $P(B)=0.5$ and $P(A \cup B)=0.7$ then find $P(A \cap B)$.
2. A die is rolled until a six appears. What is the expected number of rolls required?
3. Let X and Y be continuous random variables with joint PDF $f(x,y)=4xy$, for $0 < x < 1$, $0 < y < 1$. Are X and Y independent? Justify.
4. If the joint PMF of X and Y is given by $P(X=x, Y=y)=k(x+y)$, for $x = 1, 2$, $y = 1, 2$, find the value of k .
5. Define strictly stationary and wide-sense stationary random processes.
6. Write the transition probability matrix for Fig. 1

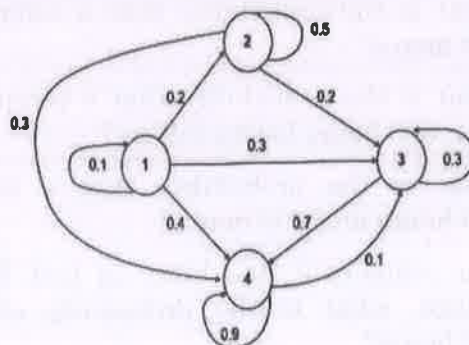


Fig. 1

7. Find the auto correlation function whose spectral density is $s(\omega) = \begin{cases} \pi, & |\omega| \\ 0, & \text{otherwise} \end{cases}$
8. A random process is given by $X(t) = A \sin(2\pi ft)$, where A is a random variable with mean zero. Find $E[X(t)]$.
9. Define cross power spectral density function.
10. State the condition for linear time invariant (LTI) system to be stable in terms of its transfer function.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A manufacturing company has three suppliers A, B, and C. The probabilities of getting a component from them are 0.3, 0.4, and 0.3 respectively. The probabilities that a component from A, B, and C is defective are 0.01, 0.02, and 0.03 respectively.
 - (1) What is the probability that a randomly selected component is defective?
 - (2) If a component is found to be defective, what is the probability it came from supplier B? (4+4)
- (ii) Suppose calls arrive at a help desk according to a Poisson process with a mean rate of 4 per hour.
 - (1) Find the probability that exactly 3 calls arrive in one hour.
 - (2) Find the probability that at least one call arrives in one hour.
 - (3) What is the probability that no call arrives in 30 minutes? (2+3+3)

Or

- (b) (i) The lifetime of an electronic component is exponentially distributed with a mean of 400 hours.
 - (1) What is the probability that a component fails in less than 100 hours?
 - (2) What is the probability that a component operates for more than 500 hours before failure?
 - (3) What is the probability that a component fails between 100 hours and 200 hours?
 - (4) If a component has been on test for 400 hours without a failure, what is the probability of a failure in the next 100 hours? (2+2+2+2)

- (ii) Assume the height of students in a class follows a normal distribution with mean 165 cm and standard deviation 10 cm.

- (1) What proportion of students are taller than 180 cm?
- (2) What proportion are between 160 cm and 170 cm?
- (3) What is the height below which 5% of the students fall?

(2+2+4)

12. (a) Let the joint pdf of random variables X and Y be given by

$$f_{xy}(x, y) = \begin{cases} 6(1-y), & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the following :

- (i) Marginal pdfs of X and Y
- (ii) $E[X]$, $E[Y]$, $Var[X]$, $Var[Y]$
- (iii) Correlation coefficient ρ_{XY}
- (iv) Are X and Y independent? Justify.

(4+6+4+2)

Or

- (b) The following table gives the data on number of hours studied (X) and score in test (Y) for 10 students:

X	1	2	3	4	5	6	7	8	9	10
Y	52	53	55	56	59	60	61	63	64	65

Find the following

- (i) The regression line of Y on X and X on Y
- (ii) Coefficient of correlation
- (iii) Estimate score (Y) when $X = 7.5$

(10+4+2)

13. (a) (i) Define Poisson process. Show that the inter arrival time in Poisson process follows an exponential distribution. (6)
- (ii) A device is monitored continuously. The probability of failure in any 1-hour window is proportional to the time elapsed. If failures occur according to a Poisson process with mean 2 per hour, compute:
- (1) Probability that no failure occurs in 30 minutes
 - (2) Probability of at least one failure in 2 hours
 - (3) Expected number of failures in 5 hours

(4+3+3)

Or

- (b) A delivery truck operates daily in three zones: Zone A, Zone B, and Zone C. On any given day, when the truck starts in Zone A, the probability that the next, delivery is also in Zone A is 0.4, in Zone B is 0.3, and in Zone C is 0.3. When in Zone B, the probability of the next delivery being in Zone A is 0.2, in Zone B is 0.5, and in Zone C is 0.3. When in Zone C, the probability of the next delivery being in Zone A is 0.3, in Zone B is 0.3, and in Zone C is 0.4.

- (i) Draw the state-transition diagram representing this Markov process.
- (ii) Write the one-step transition probability matrix.
- (iii) Determine the limiting-state probabilities, assuming they exist.

(3+3+10)

14. (a) (i) Show that the random process $X(t) = A \cos(\omega_0 t + \theta)$ is wide sense stationary if it is assumed that A and ω_0 are constants and θ is a uniformly distributed random variable on the interval $(0, 2\pi)$. (8)
- (ii) Let two random processes $X(t)$ and $Y(t)$ be defined by $X(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t)$, $Y(t) = B \cos(\omega_0 t) - A \sin(\omega_0 t)$ where A and B random variables and ω_0 is a constant. Show that the random processes $X(t)$ and $Y(t)$ are jointly wide sense stationary. (8)

Or

- (b) (i) The power spectral density is given by $S_{XX}(\omega) = \frac{\omega^2 + 9}{\omega^4 + 5\omega^2 + 4}$. Find the mean square value of the process. (8)
- (ii) The cross-power spectrum of real random processes $x(t)$ and $y(t)$ is given by $S_{xy}(\omega) = \begin{cases} a + \frac{jbw}{\omega}, & |\omega| \leq 1 \\ 0, & \text{elsewhere} \end{cases}$. Find the cross-correlation. (8)

15. (a) A system has impulse response $h(t) = e^{-2t}$, $t \geq 0$ and input $X(t)$ is a stationary process with autocorrelation $R_{xx}(\tau) = e^{-|\tau|}$.
- (i) Find $R_{XY}(\tau)$, the cross-correlation between input and output.
- (ii) Find $R_{YY}(\tau)$, the auto correlation of the output.
- (iii) Find $S_{YY}(\omega)$, the power spectral density of the output. (6+6+4)

Or

- (b) Let $X(t)$ be a Wide-Sense Stationary (WSS) random process that serves as the input to a linear time-invariant (LTI) system characterized by the frequency response $H(\omega) = \frac{1}{\alpha + j\omega}$, $\alpha > 0$. Assume that $X(t)$ is a zero-mean white noise process with a power spectral density (PSD) given by $S_X(\omega) = \frac{N_0}{2}$. Determine the following
- (i) The impulse response $h(t)$ of the system.
- (ii) The cross-power spectral density $S_{XY}(\omega)$ between the input $X(t)$ and the output $Y(t)$.
- (iii) The cross-correlation function $R_{YX}(\tau)$ of the output $Y(t)$ with the input $X(t)$.
- (iv) The power spectral density $S_{YY}(\omega)$ of the output process $Y(t)$. (4+4+4+4)